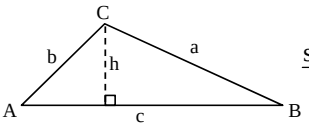
Lesson 8-5

Objective – To use the Law of Sines and the Law of Cosines to solve triangles.



Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Why?

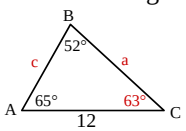
$\sin A = \frac{h}{b}$ $\quad$ $\sin B = \frac{h}{a}$

$h = b \sin A$ $\quad$ $h = a \sin B$

$b \sin A = a \sin B$

$\therefore \ \frac{\sin A}{a} = \frac{\sin B}{b}$

Solve the triangle below.



$m\angle A = 65°$
$m\angle B = 52°$
$m\angle C = 180° - 65° - 52° = 63°$

$AC = 12$
$AB \approx 13.6$
$BC \approx 13.8$

$(c)\ \frac{\sin 52°}{12} = \frac{\sin 63°}{c}(c)$

$c\left(\frac{\sin 52°}{12}\right) = \sin 63°$

$c = \sin 63°\left(\frac{12}{\sin 52°}\right)$

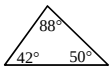
$c \approx 13.6$

$\frac{\sin 52°}{12} = \frac{\sin 65°}{a}$

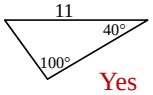
$a = \sin 65°\left(\frac{12}{\sin 52°}\right)$

$a \approx 13.8$

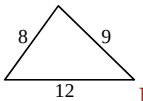
State whether you can use the Law of Sines to solve the triangle.

1) 

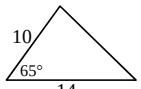
No

2) 

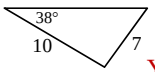
Yes

3) 

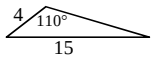
No

4) 

No

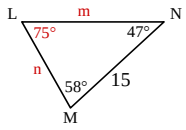
5) 

Yes

6) 

Yes

Solve the triangle below.



$m\angle M = 58°$
$m\angle N = 47°$
$m\angle L = 180° - 58° - 47° = 75°$

$MN = 15$
$LM \approx 11.4$
$LN \approx 13.2$

$(n)\ \frac{\sin 75°}{15} = \frac{\sin 47°}{n}(n)$

$n\left(\frac{\sin 75°}{15}\right) = \sin 47°$

$n = \sin 47°\left(\frac{15}{\sin 75°}\right)$

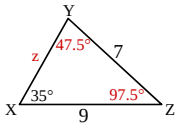
$n \approx 11.4$

$\frac{\sin 75°}{15} = \frac{\sin 58°}{m}$

$m = \sin 58°\left(\frac{15}{\sin 75°}\right)$

$m \approx 13.2$

Solve the triangle below.



$m\angle X = 35°$
$m\angle Y = 47.5°$
$m\angle Z = 180° - 35° - 47.5° = 97.5°$

$XZ = 9$
$YZ = 7$
$XY \approx 12.1$

$(9)\ \frac{\sin Y}{9} = \frac{\sin 35°}{7}(9)$

$\sin Y = 9\left(\frac{\sin 35°}{7}\right)$

$\sin Y \approx 0.7375$

$Y \approx \sin^{-1}(0.7375)$

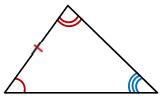
$m\angle Y \approx 47.5°$

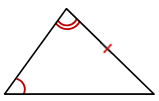
$(Z)\ \frac{\sin 97.5°}{Z} = \frac{\sin 35°}{7}(Z)$

$Z = \sin 97.5°\left(\frac{7}{\sin 35°}\right)$

$Z \approx 12.1$

When can Law of Sines be used?

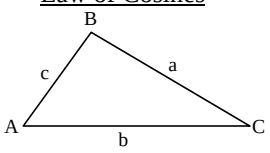

ASA


AAS


SSA (sometimes)

Lesson 8-5

Law of Cosines

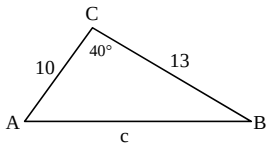


$$a^2 = b^2 + c^2 - 2bc(\cos A)$$

$$b^2 = a^2 + c^2 - 2ac(\cos B)$$

$$c^2 = a^2 + b^2 - 2ab(\cos C)$$

Find AB.

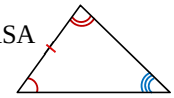


$$c^2 = a^2 + b^2 - 2ab(\cos C)$$
$$c^2 = 13^2 + 10^2 - 2(13)(10)(\cos 40°)$$
$$c^2 \approx 169 + 100 - 260(0.7660)$$
$$c^2 \approx 269 - 199.16$$
$$c^2 \approx 69.84$$
$$c \approx \sqrt{69.84} \approx 8.36$$

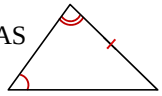
When can Law of Cosines be used?

Law of Sines

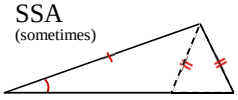
ASA



AAS

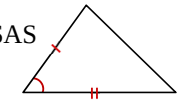


SSA
(sometimes)

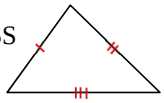


Law of Cosines

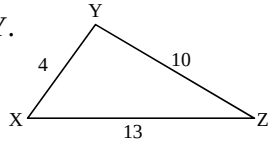
SAS



SSS



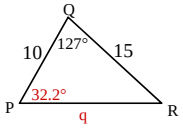
Find m∠Y.



$$y^2 = x^2 + z^2 - 2xz(\cos Y)$$
$$13^2 = 10^2 + 4^2 - 2(10)(4)(\cos Y)$$
$$169 = 100 + 16 - 80(\cos Y)$$
$$169 = 116 - 80(\cos Y)$$
$$53 = -80(\cos Y)$$
$$\frac{53}{-80} = \cos Y \approx -0.6625$$

$$Y \approx \cos^{-1}(-0.6625)$$
$$m\angle Y \approx 131.5°$$

Solve the triangle below.



PQ = 10
QR = 15
PR ≈ 22.48
m∠Q ≈ 127°
m∠P ≈ 32.2°
m∠R ≈ 180° − 127° − 32.2° ≈ 20.8°

$$q^2 = p^2 + r^2 - 2pr(\cos Q)$$
$$q^2 = 15^2 + 10^2 - 2 \cdot 15 \cdot 10(\cos 127°)$$
$$q^2 = 225 + 100 - 300(\cos 127°)$$
$$q^2 \approx 325 - 300(-0.6018)$$
$$q^2 \approx 325 + 180.54$$
$$q \approx \sqrt{505.54} \approx 22.48$$

$$\frac{\sin 127°}{22.48} = \frac{\sin P}{15}$$
$$P = \sin^{-1} 15\left(\frac{\sin 127°}{22.48}\right)$$
$$P \approx \sin^{-1}(0.5329)$$
$$m\angle P \approx 32.2°$$