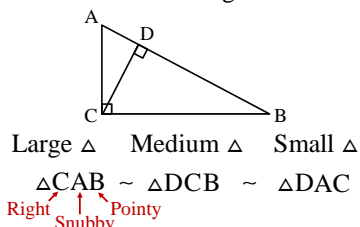


# Lesson 8-1

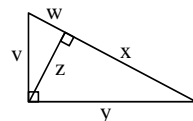
Objective – To use similarity relationships in right triangles to solve problems.

## Right Triangle Similarity Theorem

The altitude to the hypotenuse of a right triangle forms two smaller right triangles that are similar to each other and to the original.



If the  $\triangle$ s are  $\sim$ , then the sides are proportional.



	Small $\triangle$	Medium $\triangle$	Large $\triangle$
$\frac{\text{short leg}}{\text{hyp.}}$	$\frac{w}{v}$	$= \frac{z}{y}$	$= \frac{v}{w+x}$
$\frac{\text{long leg}}{\text{hyp.}}$	$\frac{z}{v}$	$= \frac{x}{y}$	$= \frac{y}{w+x}$
$\frac{\text{short leg}}{\text{long leg}}$	$\frac{w}{z}$	$= \frac{z}{x}$	$= \frac{v}{y}$

Find x, y, and z.

Small  $\triangle$     Large  $\triangle$

$$\frac{\text{short leg}}{\text{hyp.}} \quad \frac{x}{6} = \frac{6}{10}$$

$$10x = 36$$

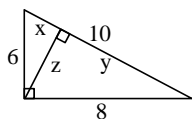
$$x = 3.6$$

Small  $\triangle$     Large  $\triangle$

$$\frac{\text{long leg}}{\text{hyp.}} \quad \frac{z}{6} = \frac{8}{10}$$

$$10z = 48$$

$$z = 4.8$$



Medium  $\triangle$     Large  $\triangle$

$$\frac{\text{long leg}}{\text{hyp.}} \quad \frac{y}{8} = \frac{8}{10}$$

$$10y = 64$$

$$y = 6.4$$

Check:

$$x + y = 10$$

$$3.6 + 6.4 = 10 \quad \checkmark$$

Find a, b, and c.

Small  $\triangle$     Large  $\triangle$

$$\frac{\text{short leg}}{\text{hyp.}} \quad \frac{a}{5} = \frac{5}{13}$$

$$13a = 25$$

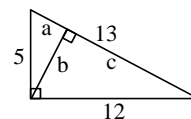
$$a = \frac{25}{13} = 1\frac{12}{13}$$

Medium  $\triangle$     Large  $\triangle$

$$\frac{\text{short leg}}{\text{hyp.}} \quad \frac{b}{12} = \frac{5}{13}$$

$$13b = 60$$

$$b = \frac{60}{13} = 4\frac{8}{13}$$



Medium  $\triangle$     Large  $\triangle$

$$\frac{\text{long leg}}{\text{hyp.}} \quad \frac{c}{12} = \frac{12}{13}$$

$$13c = 144$$

$$c = \frac{144}{13} = 11\frac{1}{13}$$

Check:

$$a + c = 13$$

$$1\frac{12}{13} + 11\frac{1}{13} = 13 \quad \checkmark$$

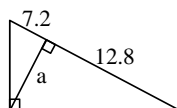
## Special Situation #1

Small  $\triangle$     Medium  $\triangle$

$$\frac{\text{long leg}}{\text{short leg}} \quad \frac{a}{7.2} = \frac{12.8}{a}$$

$$a^2 = 92.16$$

$$a = \sqrt{92.16} = 9.6$$



Geometric Mean - the positive square root of the product of two given numbers.

## Altitude is Geometric Means Corollary

The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the two segments of the hypotenuse.

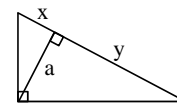
## Geometric Means Corollary

The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the two segments of the hypotenuse.

$$\frac{\text{long leg}}{\text{short leg}} \quad \frac{a}{x} = \frac{y}{a}$$

$$a^2 = xy$$

$$a = \sqrt{xy}$$



Find the geometric mean of the numbers below.

1) 9    16

$$a = \sqrt{9 \cdot 16}$$

$$a = \sqrt{144} = 12$$

2) 5    8

$$a = \sqrt{5 \cdot 8}$$

$$a = \sqrt{40} = 2\sqrt{10}$$

# Lesson 8-1

**Special Situation #2**

<p>Small <math>\Delta</math> Large <math>\Delta</math></p> $\frac{\text{short leg}}{\text{hyp.}} = \frac{7.2}{x} = \frac{x}{20}$ $x^2 = 144$ $x = \sqrt{144}$ $x = 12$		<p>Medium <math>\Delta</math> Large <math>\Delta</math></p> $\frac{\text{long leg}}{\text{hyp.}} = \frac{12.8}{y} = \frac{y}{20}$ $y^2 = 256$ $y = \sqrt{256}$ $y = 16$
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**Legs are Geometric Means Corollary**

The length of a leg of a right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

Write a similarity statement comparing the three triangles in each diagram.

1)

	<p>Large <math>\Delta</math>    Medium <math>\Delta</math>    Small <math>\Delta</math></p> <p><math>\Delta LMN \sim \Delta NMP \sim \Delta LNP</math></p> <p>Snubby Pointy Right</p>
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2)

	<p>Large <math>\Delta</math>    Medium <math>\Delta</math>    Small <math>\Delta</math></p> <p><math>\Delta HIJ \sim \Delta KIH \sim \Delta KHJ</math></p> <p>Right Pointy Snubby</p>
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Find a, b, and c.

<p>Small <math>\Delta</math> Large <math>\Delta</math></p> $\frac{\text{hyp.}}{\text{short leg}} = \frac{a}{5} = \frac{12}{a}$ $a^2 = 60$ $a = \sqrt{60} = 2\sqrt{15}$		<p>Small <math>\Delta</math> Medium <math>\Delta</math></p> $\frac{\text{long leg}}{\text{short leg}} = \frac{b}{5} = \frac{7}{b}$ $b^2 = 35$ $b = \sqrt{35}$
<p>Medium <math>\Delta</math> Large <math>\Delta</math></p> $\frac{\text{hyp.}}{\text{long leg}} = \frac{c}{7} = \frac{12}{c}$ $c^2 = 84$ $c = \sqrt{84} = 2\sqrt{21}$		

Find x, y, and z.

<p>Large <math>\Delta</math> Medium <math>\Delta</math></p> $\frac{\text{hyp.}}{\text{long leg}} = \frac{h}{12} = \frac{12}{8}$ $8h = 144$ $h = 18$ $\therefore z + 8 = 18$ $\therefore z = 10$	<p>Medium <math>\Delta</math></p> $a^2 + b^2 = c^2$ $8^2 + y^2 = 12^2$ $64 + y^2 = 144$ $y^2 = 80$ $y = \sqrt{80} = 4\sqrt{5}$
	<p>Small <math>\Delta</math> Large <math>\Delta</math></p> $\frac{\text{hyp.}}{\text{short leg}} = \frac{x}{10} = \frac{18}{x}$ $x^2 = 180$ $x = \sqrt{180} = 6\sqrt{5}$

Find x.

	<p>Large <math>\Delta</math>    Medium <math>\Delta</math>    Small <math>\Delta</math></p> <p>hyp?    long leg?    long leg</p> <p>short leg    short leg    short leg</p>	
<p>Small <math>\Delta</math> Medium <math>\Delta</math></p> $\frac{\text{short leg}}{\text{long leg}} = \frac{5}{7} = \frac{7}{x}$ $5x = 49$ $x = \frac{49}{5} = 9\frac{4}{5}$		

Find x.

	<p>Small <math>\Delta</math>    Medium <math>\Delta</math>    Large <math>\Delta</math></p> <p>short leg    long leg?    long leg</p> <p>hyp.    hyp.    hyp?</p>	
<p>Medium <math>\Delta</math> Large <math>\Delta</math></p> $\frac{\text{long leg}}{\text{hyp.}} = \frac{x}{9} = \frac{9}{x+4}$ $x(x+4) = 81$ $x^2 + 4x - 81 = 0$ $(x + \quad)(x - \quad) = 0$ <p style="text-align: center; font-size: small;">1•81 3•27 9•9</p>	$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-81)}}{2(1)}$ $x = \frac{-4 \pm \sqrt{340}}{2} = \frac{-4 \pm 2\sqrt{85}}{2}$ $-2 \pm \sqrt{85} \text{ or } -2 + \sqrt{85}$ $x = -2 + \sqrt{85} \approx 7.22$	