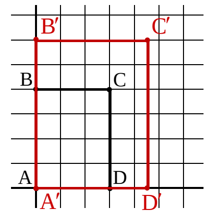
Lesson 7-6

Objective – To apply properties of similarity to dilations in the coordinate plane.

Dilation - A transformation that changes the size of a figure using a scale factor.

Draw the image of Rectangle ABCD using a dilation $k = \frac{3}{2}$ about the origin.

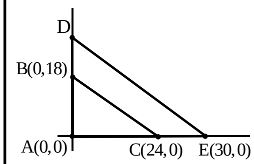


$A(0,0) \times \frac{3}{2} = A'(0,0)$

$B(0,4) \times \frac{3}{2} = B'(0,6)$

$C(3,4) \times \frac{3}{2} = C'\left(\frac{9}{2},6\right)$

$D(3,0) \times \frac{3}{2} = D'\left(\frac{9}{2},0\right)$



Given $\triangle ABC \sim \triangle ADE$, find the scale factor of the dilation, k, and the coordinates of point D.

$\frac{AB}{AD} = \frac{AC}{AE}$

$\frac{18}{x} = \frac{24}{30}$

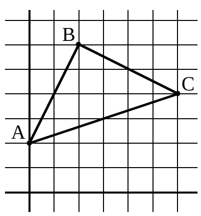
$\frac{540}{24} = \frac{24x}{24}$

$22.5 = x$

$AD = 22.5 \therefore \boxed{D = (0, 22.5)}$

If $\triangle ABC$ is the pre-image, then $k = \frac{30}{24} = \boxed{\frac{5}{4}}$

If $\triangle ADE$ is the pre-image, then $k = \frac{24}{30} = \boxed{\frac{4}{5}}$



A(0,2) B(2,6) C(6,4)
Q(0,9) R(x,y) S(27,18)

Given $\triangle ABC \sim \triangle QRS$, find the scale factor of the dilation, k, and the coordinates of point R, the image of B.

Since $\triangle QRS$ is the image of $\triangle ABC$,

$k = \frac{QS}{AC} = \frac{9\sqrt{10}}{2\sqrt{10}} = \boxed{\frac{9}{2}}$

$QS = \sqrt{(18-9)^2 + (27-0)^2}$

$QS = \sqrt{81+729} = \sqrt{810} = 9\sqrt{10}$

$AC = \sqrt{(4-2)^2 + (6-0)^2}$

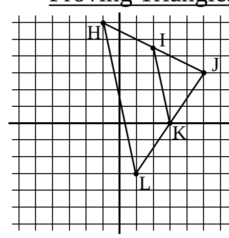
$AC = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$

$\therefore B(2,6) \times k = R(x,y)$

$\therefore B(2,6) \times \frac{9}{2} = R\left(2 \cdot \frac{9}{2}, 6 \cdot \frac{9}{2}\right)$

$= \boxed{R(9,27)}$

Proving Triangles Similar in Coordinate Plane



Given:
H(–1,6) I(2,4.5) J(5,3)
K(3,0) L(1,-3)

Prove: $\triangle IJK \sim \triangle HJL$

$KJ = \sqrt{(0-3)^2 + (3-5)^2}$

$KJ = \sqrt{9+4} = \sqrt{13}$

$LJ = \sqrt{(-3-3)^2 + (1-5)^2}$

$LJ = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

$IJ = \sqrt{(3-4.5)^2 + (5-2)^2} = \sqrt{2.25+9}$

$IJ = \sqrt{11\frac{1}{4}} = \sqrt{\frac{45}{4}} = \frac{\sqrt{45}}{\sqrt{4}} = \frac{3\sqrt{5}}{2}$

$HJ = \sqrt{(3-6)^2 + (5--1)^2}$

$HJ = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$

$\frac{IJ}{HJ} = \frac{\frac{3\sqrt{5}}{2}}{3\sqrt{5}} = \boxed{\frac{1}{2}}$ $\quad \frac{KJ}{LJ} = \frac{\sqrt{13}}{2\sqrt{13}} = \boxed{\frac{1}{2}}$

Since $\frac{IJ}{HJ} = \frac{KJ}{LJ}$ and $\angle J \cong \angle J$,

$\therefore \triangle IJK \sim HLJ$ by SAS Similarity.

Proving Triangles Similar in Coordinate Plane

Given:
A(0,2) B(4,0) C(6,4)
L(0,3) M(6,0) N(9,6)

Prove: $\triangle ABC \sim \triangle LMN$

$BC = \sqrt{(4-0)^2 + (6-4)^2}$

$BC = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

$MN = \sqrt{(6-0)^2 + (9-6)^2}$

$MN = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$

$\frac{BC}{MN} = \frac{2\sqrt{5}}{3\sqrt{5}} = \boxed{\frac{2}{3}}$

$\frac{AB}{LM} = \frac{AC}{LN} = \frac{BC}{MN}$

$\therefore \triangle ABC \sim \triangle LMN$ by SSS Similarity.

$AB = \sqrt{(0-2)^2 + (4-0)^2}$

$AB = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$LM = \sqrt{(0-3)^2 + (6-0)^2}$

$LM = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$

$\frac{AB}{LM} = \frac{2\sqrt{5}}{3\sqrt{5}} = \boxed{\frac{2}{3}}$

$AC = \sqrt{(4-2)^2 + (6-0)^2}$

$AC = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$

$LN = \sqrt{(6-3)^2 + (9-0)^2}$

$LN = \sqrt{9+81} = \sqrt{90} = 3\sqrt{10}$

$\frac{AC}{LN} = \frac{2\sqrt{10}}{3\sqrt{10}} = \boxed{\frac{2}{3}}$