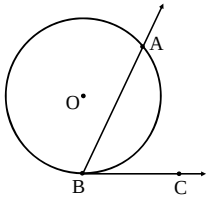
Lesson 11-5

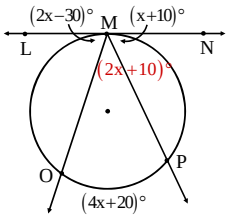
Objective – To find the measure of angles formed by secants and tangents.

If a tangent and a secant intersect a circle at the point of tangency, then the angle formed is half the measure of the intercepted arc.



$$m\angle ABC = \frac{1}{2}m\overset{\frown}{AB}$$

Find $m\overset{\frown}{LO}$.



$$m\angle OMP = \frac{1}{2}m\overset{\frown}{OP}$$

$$m\angle OMP = \frac{1}{2}(4x+20)°$$

$$m\angle OMP = (2x+10)°$$

$$(2x-30)° + (2x+10)° + (x+10)° = 180°$$

$$5x - 10 = 180$$

$$5x = 190$$

$$x = 38$$

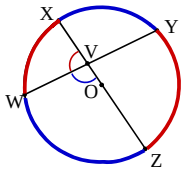
$$m\angle LMO = \frac{1}{2}m\overset{\frown}{MO}$$

$$(2 \cdot 38 - 30)° = \frac{1}{2}m\overset{\frown}{MO}$$

$$2(46)° = m\overset{\frown}{MO} = 92°$$

Interior Intersection of Two Chords Theorem

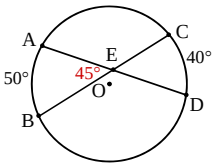
If two chords or secants intersect in the interior of a circle, then the measure of each angle formed is half the sum of the measures of the intercepted arcs.



$$m\angle XVW = \frac{1}{2}\left(m\overset{\frown}{WX} + m\overset{\frown}{YZ}\right)$$

$$m\angle WVZ = \frac{1}{2}\left(m\overset{\frown}{XY} + m\overset{\frown}{WZ}\right)$$

Find $m\angle AEC$.



$$m\angle AEB = \frac{1}{2}\left(m\overset{\frown}{AB} + m\overset{\frown}{CD}\right)$$

$$m\angle AEB = \frac{1}{2}(50° + 40°)$$

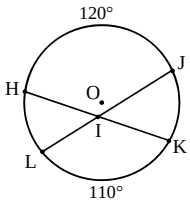
$$m\angle AEB = 45°$$

$$m\angle AEB + m\angle AEC = 180°$$

$$45° + m\angle AEC = 180°$$

$$m\angle AEC = 135°$$

Find $m\angle HIL$.



$$m\angle HIJ = \frac{1}{2}\left(m\overset{\frown}{HJ} + m\overset{\frown}{KL}\right)$$

$$m\angle HIJ = \frac{1}{2}(120° + 110°)$$

$$m\angle HIJ = 115°$$

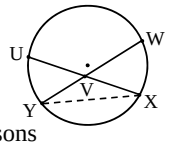
$$m\angle HIJ + m\angle HIL = 180°$$

$$115° + m\angle HIL = 180°$$

$$m\angle HIL = 65°$$

Given: $\overline{UX}$ and $\overline{WY}$ intersect at V

Prove: $m\angle UVY = \frac{1}{2}\left(m\overset{\frown}{UY} + m\overset{\frown}{WX}\right)$



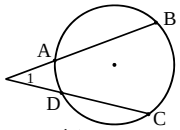
Statement	Reasons
1) $\overline{UX}$ and $\overline{WY}$ intersect at V	Given
2) Draw auxillary line $\overline{XY}$	Two pts. determine a line
3) $m\angle UVY = m\angle VXY + m\angle VYX$	Ext. $m\angle$ = Sum of Remote Interior $\angle$s Theorem
4) $m\angle VXY = \frac{1}{2}m\overset{\frown}{UY}$, $m\angle VYX = \frac{1}{2}m\overset{\frown}{WX}$	Inscribed $\angle$ Thm.
5) $m\angle UVY = \frac{1}{2}m\overset{\frown}{UY} + \frac{1}{2}m\overset{\frown}{WX}$	Substitution
6) $m\angle UVY = \frac{1}{2}\left(m\overset{\frown}{UY} + m\overset{\frown}{WX}\right)$	Distributive Prop.

Lesson 11-5

Exterior Intersection of Two Secants Theorem

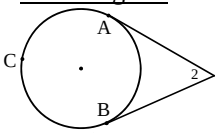
If two tangents or secants intersect on the exterior of a circle, then the measure of the angle formed is half the difference of the intercepted arc measures.

Two Secants



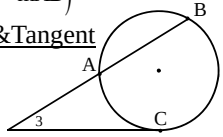
$$m\angle 1 = \frac{1}{2}\left(m\overset{\frown}{BC} - m\overset{\frown}{AD}\right)$$

Two Tangents



$$m\angle 2 = \frac{1}{2}\left(m\overset{\frown}{ACB} - m\overset{\frown}{AB}\right)$$

Secant &Tangent



$$m\angle 3 = \frac{1}{2}\left(m\overset{\frown}{BC} - m\overset{\frown}{AC}\right)$$

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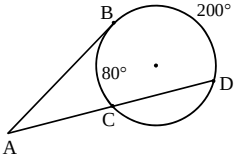
1) Find $m\angle A$.

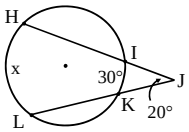


$$m\angle A = \frac{1}{2}\left(m\overset{\frown}{BD} - m\overset{\frown}{BC}\right)$$

$$m\angle A = \frac{1}{2}\left(200° - 80°\right)$$

$$m\angle A = 60°$$

2) Find $m\overset{\frown}{HL}$.



$$m\angle J = \frac{1}{2}\left(m\overset{\frown}{HL} - m\overset{\frown}{IK}\right)$$

$$20° = \frac{1}{2}\left(x - 30°\right)$$

$$40° = x - 30°$$

$$70° = x$$

$$m\overset{\frown}{HL} = 70°$$

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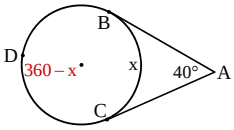
3) Find $m\overset{\frown}{BC}$.



$$m\angle A = \frac{1}{2}\left(m\overset{\frown}{BDC} - m\overset{\frown}{BC}\right)$$

$$40° = \frac{1}{2}\left(360° - x - x\right)$$

$$40° = 180° - x$$

$$140° = x$$

$$m\overset{\frown}{BC} = 140°$$

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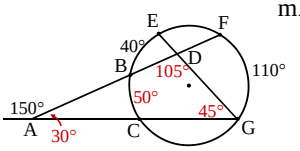
4) Find $m\angle EDF$.



$$m\angle ADG = 180° - 30° - 45°$$

$$m\angle ADG = 105°$$

$$m\angle EDF = m\angle ADG$$

$$m\angle EDF = 105°$$

$$m\angle BAC = \frac{1}{2}\left(m\overset{\frown}{FG} - m\overset{\frown}{BC}\right)$$

$$30° = \frac{1}{2}\left(110° - m\overset{\frown}{BC}\right)$$

$$60° = 110° - m\overset{\frown}{BC}$$

$$50° = m\overset{\frown}{BC}$$

$$m\angle G = \frac{1}{2}\left(m\overset{\frown}{BE} + m\overset{\frown}{BC}\right)$$

$$m\angle G = \frac{1}{2}\left(40° + 50°\right)$$

$$m\angle G = 45°$$