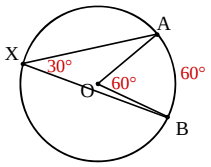
Lesson 11-4

Objective – To find the measure of inscribed angles and use them to solve problems.

Inscribed Angle - an angle whose vertex lies on the circle and whose sides are chords of the circle.



$\angle AOB$ is a central angle.

$\angle AXB$ is an inscribed angle.

$\angle X$ intercepts $\overset{\frown}{AB}$.

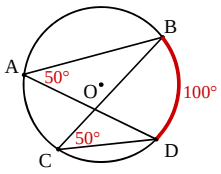
$\overset{\frown}{AB}$ subtends $\angle X$.

Inscribed Angle Theorem

The measure of an inscribed angle is half the measure of its intercepted arc.

$$m\angle X = \frac{1}{2} m\overset{\frown}{AB}$$

If $m\angle BAD = 50°$, find $m\angle BCD$.



$\angle A$ intercepts $\overset{\frown}{BD}$

$\therefore m\overset{\frown}{BD} = 100°$

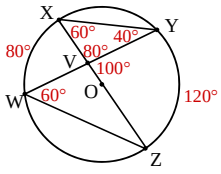
$\overset{\frown}{BD}$ subtends $\angle C$

$\therefore m\angle BCD = 50°$

Inscribed Angle Corollary

If inscribed angles of a circle intercept the same arc, then the angles are congruent or are subtended by the same chord.

Find the following if $m\angle X = 60°$ and $m\overset{\frown}{WX} = 80°$.



1) $m\overset{\frown}{YZ} = 120°$

2) $m\angle W = 60°$

3) $m\angle Y = 40°$

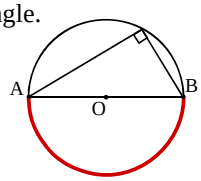
4) $m\angle XVY = 80°$

5) $m\angle YVZ = 100°$

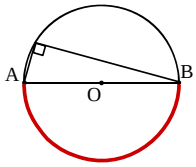
Inscribed Right Triangle Theorem

An inscribed angle intercepts a semicircle if and only if the angle is a right angle.

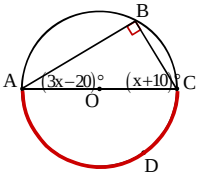
If an inscribed angle intercepts a semicircle then the angle is a right angle.



If an inscribed angle is a right angle then it intercepts a semicircle .



Find $m\overset{\frown}{BC}$.



$\angle B$ intercepts $\overset{\frown}{ADC}$ which is a semicircle.

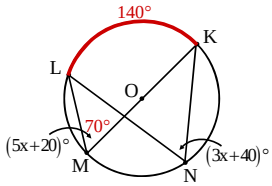
$\therefore m\angle B = 90°$

$\therefore \angle A$ and $\angle C$ are complementary.

$$(3x-20)° + (x+10)° = 90°$$
$$4x - 10 = 90$$
$$4x = 100$$
$$x = 25$$

$$m\angle A = 3x - 20$$
$$m\angle A = 3(25) - 20$$
$$m\angle A = 55°$$
$$m\overset{\frown}{BC} = 2m\angle A = 2(55°)$$
$$m\overset{\frown}{BC} = 110°$$

Find $m\overset{\frown}{ML}$.



$\angle M$ and $\angle N$ intercept $\overset{\frown}{LK}$.

$\therefore m\angle M = m\angle N$

$$(5x+20)° = (3x+40)°$$
$$2x = 20$$
$$x = 10$$
$$m\angle M = (5x+20)° = 70°$$
$$\therefore m\overset{\frown}{LK} = 140°$$

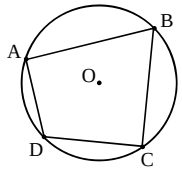
Since $\overline{MK}$ is a diameter, $\overset{\frown}{MLK}$ is a semicircle.

$$\therefore m\overset{\frown}{ML} + m\overset{\frown}{LK} = 180°$$
$$m\overset{\frown}{ML} + 140° = 180°$$
$$m\overset{\frown}{ML} = 40°$$

Lesson 11-4

Inscribed Quadrilateral Theorem

If a quadrilateral is inscribed in a circle then its opposite angles are supplementary.

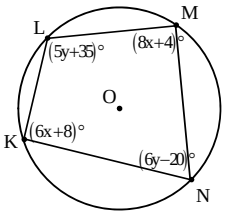


Supplementary Angles

$m\angle A + m\angle C = 180°$

$m\angle B + m\angle D = 180°$

Find the angles of the inscribed quadrilateral.



$(6x+8)° + (8x+4)° = 180°$

$14x + 12 = 180$

$14x = 168$

$x = 12$

$(5y+35)° + (6y-20)° = 180°$

$11y + 15 = 180$

$11y = 165$

$y = 15$

$m\angle K = 6(12) + 8 = 80°$

$m\angle M = 8(12) + 4 = 100°$

$m\angle L = 5(15) + 35 = 110°$

$m\angle N = 6(15) - 20 = 70°$