

Lesson 4-5

Objective - To find and use inverse matrices.

Inverse of a 2×2 Matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$A^{-1} = \frac{1}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$ad - cb \neq 0$$

Find the inverse of A

$$A = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}} \begin{bmatrix} 5 & -4 \\ -3 & 2 \end{bmatrix} = \frac{1}{10-12} \begin{bmatrix} 5 & -4 \\ -3 & 2 \end{bmatrix}$$

$$= -\frac{1}{2} \begin{bmatrix} 5 & -4 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -\frac{5}{2} & 2 \\ -\frac{3}{2} & -1 \end{bmatrix}$$

Find the inverse of A

$$A = \begin{bmatrix} -3 & 2 \\ -8 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\begin{bmatrix} -3 & 2 \\ -8 & 1 \end{bmatrix}} \begin{bmatrix} 1 & 2 \\ -8 & -3 \end{bmatrix} = \frac{1}{-3 - (-16)} \begin{bmatrix} 1 & 2 \\ -8 & -3 \end{bmatrix}$$

$$= \frac{1}{13} \begin{bmatrix} 1 & 2 \\ -8 & -3 \end{bmatrix} = \begin{bmatrix} \frac{1}{13} & \frac{2}{13} \\ -\frac{8}{13} & -\frac{3}{13} \end{bmatrix}$$

Find A^{-1} and multiply $A \times A^{-1}$

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 6 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} -3 & 2 \\ 1 & -\frac{1}{2} \end{bmatrix}$$

$$A \times A^{-1} = \begin{bmatrix} 1 & 4 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 1 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} -3+4 & 2-2 \\ -6+6 & 4-3 \end{bmatrix}$$

$$A \times A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Identity Matrix

Solving a Matrix Equation

$$\begin{bmatrix} 3 & 9 \\ 2 & 7 \end{bmatrix} X = \begin{bmatrix} 1 & 5 \\ 4 & 7 \end{bmatrix}$$

A

Find A^{-1}

$$A^{-1} = \begin{bmatrix} \frac{7}{3} & -3 \\ -\frac{2}{3} & 1 \end{bmatrix}$$

Solving a Matrix Equation

$$\begin{bmatrix} 3 & 9 \\ 2 & 7 \end{bmatrix} X = \begin{bmatrix} 1 & 5 \\ 4 & 7 \end{bmatrix}$$

$$\begin{bmatrix} \frac{7}{3} & -3 \\ -\frac{2}{3} & 1 \end{bmatrix} \begin{bmatrix} 3 & 9 \\ 2 & 7 \end{bmatrix} X = \begin{bmatrix} \frac{7}{3} & -3 \\ -\frac{2}{3} & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 4 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 7-6 & 21-21 \\ -2+2 & -6+7 \end{bmatrix} X = \begin{bmatrix} \frac{7}{3}-12 & \frac{35}{3}-21 \\ -\frac{2}{3}+4 & -\frac{10}{3}+7 \end{bmatrix}$$

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Solving a Matrix Equation

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} \frac{29}{3} & \frac{28}{3} \\ \frac{10}{3} & \frac{11}{3} \end{bmatrix}$$

Solving a Matrix Equation

$$\begin{bmatrix} 5 & 15 \\ 1 & 4 \end{bmatrix} X = \begin{bmatrix} 2 & 15 \\ 8 & 5 \end{bmatrix}$$

$$\begin{bmatrix} \frac{4}{5} & -3 \\ -\frac{1}{5} & 1 \end{bmatrix} \begin{bmatrix} 5 & 15 \\ 1 & 4 \end{bmatrix} X = \begin{bmatrix} \frac{4}{5} & -3 \\ -\frac{1}{5} & 1 \end{bmatrix} \begin{bmatrix} 2 & 15 \\ 8 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 4-3 & 12-12 \\ -1+1 & -3+4 \end{bmatrix} X = \begin{bmatrix} \frac{8}{5}-24 & 12-15 \\ -\frac{2}{5}+8 & -3+5 \end{bmatrix}$$

Solving a Matrix Equation

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} \frac{112}{5} & -3 \\ \frac{38}{5} & 2 \end{bmatrix}$$

Objective - To solve systems using inverse matrices.

Rewrite the following system using matrices.

$$4x + 6y = 1$$

$$3x + 7y = -2$$

$$\begin{bmatrix} 4 & 6 \\ 3 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Solve the following system using matrices.

$$2x - 3y = -11$$

$$5x - 2y = 0$$

Find A^{-1}

$$\begin{bmatrix} \frac{2}{11} & \frac{3}{11} \\ \frac{5}{11} & \frac{2}{11} \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -11 \\ 0 \end{bmatrix} \begin{bmatrix} \frac{2}{11} & \frac{3}{11} \\ \frac{5}{11} & \frac{2}{11} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{2}{11} & \frac{3}{11} \\ \frac{5}{11} & \frac{2}{11} \end{bmatrix} \begin{bmatrix} -11 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2+3 \\ 5+0 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

Solve the following system using matrices.

$$-3x + 2y = 1$$

$$-5x + 3y = 2$$

Find A^{-1}

$$\begin{bmatrix} 3 & -2 \\ 5 & -3 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 5 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 5 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3-4 \\ 5-6 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

Lesson 4-5 (cont.)

Solve the following system using matrices.

$$9x - 2y = 51$$

$$-4x - 8y = 4$$

Find A^{-1}

$$\begin{bmatrix} \frac{1}{10} & -\frac{1}{40} \\ -\frac{1}{20} & -\frac{9}{80} \end{bmatrix} \begin{bmatrix} 9 & -2 \\ -4 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 51 \\ 4 \end{bmatrix} \begin{bmatrix} \frac{1}{10} & -\frac{1}{40} \\ -\frac{1}{20} & -\frac{9}{80} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{10} & -\frac{1}{40} \\ -\frac{1}{20} & -\frac{9}{80} \end{bmatrix} \begin{bmatrix} 51 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$